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Adequacy of Variable Selection Techniques in Modeling the Determinants of Money Income to Small-Scale Business in Aba, Abia State Nigeria Using Multiple Linear Regression Model

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Abstract

This study investigates the adequacy of variable selection techniques in modeling the determinants of money income to small-scale businesses in Aba, Abia State, Nigeria using multiple linear regression model. Variable selection techniques are used to identify the best subset of regressors or explanatory variables to be included in a regression model. The explanatory variables used for this study are inflation rate, exchange rate, consumer price index and private sector credit. This study captured the effects of inflation rate (monthly percentage change in the CPI X_1), exchange rate (USD X_2), consumer price index (CPI X_3), and private sector credit (PSC, million of currency unit X_4) on Y_t called money income (million of currency unit). The method of ordinary least square estimation of regression was employed in the estimation of the model parameters after obtaining the descriptive statistics on table 4.1 for the data sets. The forward selection method, backward selection method, and stepwise selection method were utilized in the selection of an appropriate model for the money income generation of small-scale business man in Aba. With the application of model assessment criterion called Akaike Information Criterion, R^2 and adjusted R^2 statistic, the study identified the stepwise selection method regression model with exchange rate, private sector credit, their interactions and square as more appropriate in the modeling of the behavior of small-scale business money income generation in Aba, Abia State.

Keywords: Inflation, Exchange rate, Variable selection, Regression parameter

1.0 Introduction

Variable selection techniques are used primarily to identify the best subset of regressors or explanatory variables to be included in a regression model (Reunanen, 2003). According to Nduka (2010), the process of identifying and fitting an appropriate model from experimental data requires some use of statistical experimental design fundamentals. The best model for analysis of data is sample size dependent as smallest effect can only be revealed as sample size increases. Hence the amount of information in large data sets greatly exceeds the information in small data sets. Inferences on models with too few variables (parameters) can be biased, while models having too many variables (parameters) tend to have poor precision effect and are in fact spurious.

There is need for a balance between under-fitted and over-fitted models. This, according to Forster (2000), is the so-called model selection problem in regression model building. He went further to describe the best model as the smallest model that fits the data. It is on the strength of this argument that this study wants to investigate the best of these variables (inflation rate, exchange rate, consumer price index and the private sector credit) that best explains money income to small-scale businesses in Aba, Nigeria.

The smallest model in that context is one which is closest to the true response. This is a quality often strived for in model building. As pointed out by Anderson and Burnham (2008), three general principles guide model- based inferences in the sciences. These principles are simplicity and parsimony, several working hypothesis and strength of evidence. Simplicity and parsimony is a concept of Occam's razor as in Forster (2000), which suggests that the simplest explanation is probably the most likely. Parsimony enjoys a future place in scientific thinking in general and in modeling specifically. Parsimony is evident in the subject of model building, where the investigator must make compromise between model bias and variance. Here, bias corresponds to the difference between the estimated value and the true unknown value of the parameter, while variance reflects the precision of these estimates. A common measure of precision is standard error of the estimate (Forster, 2000).

According to Montgomery, D. C., Peck, E. A., and Vining, G. G. (2011), the principle of multiple working hypotheses consists of testing a hypothesis from one experiment and from the results,

formulating a new hypothesis to test on a new experiment. Chamberlin (1965) advocated the concept of several working hypothesis. Where there is no null hypothesis but there are several well-supported hypotheses (equivalently models) that are been considered. Relevant empirical data are then gathered, analyzed, and it is expected that the results would tend to support one or more hypotheses, while holding less support for other hypotheses. He stressed further, that repetition of this general approach leads to advances in the sciences. New or more elaborate hypotheses are added, while hypotheses with little empirical support are gradually dropped from the consideration.

An important feature of this multiplicity is that, the number of alternative models are kept small. The analysis of hundreds or thousands of models is not justified except when prediction is the only objective, or in the most explorative phases of an investigation. According to (Chernoff, 1953), analysis of models translates into testing for the set of data, a series of plausible models specified before conducting the analyses. From the analyses, there is an indication of which model is the best among those considered and a measure of strength of evidence for each model is established.

1.1 Statement of the problem

One of the assumptions of the classical linear regression model (CLRM) is that the regression model used in analysis is ‘‘correctly’’ specified. If the model is not “correctly” specified, we encounter the problem of model specification error or model specification bias. The question of how a researcher goes about finding the “correct” model for empirical analysis, becomes an issue of concern that needs to be investigated. The issue of model specification bias leads to the problem of under-fitting a model (omitting relevant variables) and over-fitting a model (including unnecessary variables).

To overcome the specification bias problem of under-fitting and over-fitting a model, we need to strike a balance between the two (Tsai, 1989). This approach requires that we include in the model only explanatory variables that, on theoretical grounds, are directly influenced by the dependent variable and that are not accounted for by other included variables. The desire to identify a model that will represent the true reality and also be consistent for prediction of money income, is the motivation for this research study.

Researchers/experimenters encounter situations where the full model may not be appropriate for a reliable result and prediction, hence require a subset of the regressors to use in the model. Variable selection or model building techniques may be used to identify the best subset of regressors to be included in a regression model. The subset of these variables may include the variables under consideration, their square and interactions. This study examined the goodness of fit of the regression models identified, which are used as representatives of the true realities and this requires using some optimality criteria and measures of adequacy.

1.2 Objectives of the study

The objectives of this study include;

1. Identify suitable model(s) using variable selection techniques to improve the money income generation of small-scale businesses in Aba, Abia State, Nigeria.
2. Establish the adequacy of the resulting model(s) that improves the money income generation (MIG) of small-scale businesses.
3. To fit the resulting model(s) and use it for prediction of income generation within a specified time frame.

1.3 Scope and limitation of the study

The study is focused on the principles of variable selection techniques in model building. Data on the Money income, Inflation rate, Exchange Rate, Consumer Price Index and Private Sector Credit were obtained from monthly records of 2020 Statistical Bulletin of Nigerian Financial Sector Report and Central Bank of Nigeria Bulletin 2020 and only three variable selection techniques are employed and standard assessment criteria are restricted to Akaike information criteria (AIC), R^2 and R^2 adjusted statistic, mean square error and predicted error sum of square.

1.4 Significance of the study

1 The model produced by the application of the principle of variable selection will be directly useful in determining the behaviour of the monthly data.

2 The knowledge gained will be useful in the design of models for any time series data of interest.

3 This research will establish that small-scale businesses has a great potential in economic sustenance for food consumption.

4 This work will establish the use of mathematical model in money income generation to help the small-scale businesses to estimate their income within a given period.

5 It can serve as a source of motivation to the Polytechnic community and Aba in general.

6 It will create job opportunities if effectively implemented.

7 It will enable a small-scale business man to make predictions and plan ahead for long term investment.

2.0 Literature Review

2.1 Variable Selection Techniques in Regression Models

Building statistical models of responses as functions of multiple explanatory variables is a common practice in various profession, eg medical sciences, engineering, biological sciences, ecology, agricultural sciences, information technology, the social sciences, etc. Such models serve a good number of purposes such as prediction of responses for current cases (Lee and Ghosh, (2019)), risk estimation (Gerritsen et al, (1996)), understanding or at least formulating hypothesis about the cause and effect relationship (Severini, (2010)), and constructing parsimonious model of special or immediate correlation of response values (Hoeting et al, (2016)), Lee and Ghosh (2019) etc.

There are many techniques available for statistical comparison of multiple variable models. They include the following: Mallow's C_p – statistic, Akaike information criteria (AIC), Schwarz's Bayesian information criteria (BIC), etc. Recently, Akaike information criteria and Schwarz's Bayesian information criteria have proven favourable among different professions (Burnham and Anderson (2001)), (Hobbs and Hilborn (2016)). Many of the application of these methods seem to be based on their independence from the hypothesis testing framework of frequentist statistics. (Pawitan, (2011)), for instance, observed that the AIC is remarkably superior on model selection than the hypothesis- based approach. This observation had the support of (Lukacs et al, (2007)).

Various methods have been developed for evaluating only a small number of subsets by either adding or deleting regressors one at a time. According to (Lee and Ghosh, (2019)), these methods are generally referred to as stepwise type procedures. They can be classified into three broad categories: (i) Forward Selection (ii) Backward Elimination and (iii) Stepwise Regression. Several authors have discussed and evaluated different methods of variable selection. For example, (Olden

and Jackson, (2002)) used variable selection method to investigate the Melanocortin 1 Receptor (MC1R) Polymorphism in several population.

They concluded that MC1R is under strong functional constraint in Africa, where any diversion from eumelanmin production (black pigmentation) appear to be evolutionary deleterious. Sauerbrei et al 2007, assessed three model selection methods consisting of Akaike's information criterion (AIC) using p-value from ANOVA, C_p - value method and stepwise procedure in their paper. Generally, results from these methods are usually compared to determine the best model. All selected models are based on the heredity principles to design the possible model for each design. They used the actual data from literature, consisting of the 2^2 and 2^3 factorial design to determine their final model. The results showed that the p-value and stepwise method gave the highest percentage of matched methods.

Hobbs and Hilborn (2016), assessed how often different variable selection techniques were applied in contemporary epidemiological analysis. It was of particular interest to see whether modern methods such as shrinkage or personalized regression were used in recent publications. They observed that stepwise selection methods remain the predominant method for variable selection in publication in epidemiological journals. (Lee and Ghosh (2019)), considered the predictive ability of statistical models obtained by applying seven methods of variable selection to 12 ecological and environmental data sets. The results showed that in most of the data sets, two methods based on the regression trees yielded models with lower predictive ability. Also they argued that there is no best method of variable selection and that any of the regression-based approaches is capable of yielding useful predictive models.

(Raffallovich et al (2008)), examined three shortcomings of stepwise regression, namely, subjective critical values, model uncertainty and parameter estimate bias. They argued that, in certain circumstances, the information theory approach may be sensitive to these issues as well. They found out areas where further testing and development could enhance the performance of information theory methods and ultimately leads to robust influence in behavioral ecology. Reunanen, (1998) for instance, found that approaches based on F-test, Mallow's C_p statistic and BIC has similar frequencies of current decisions about inclusion or exclusion of predictor variables.

According to (Hobbs and Hilborn (2016)) and (Myers, R. H. (1990)), there is need for correctly specifying the regression model as leaving out important regressors introduces bias into the parameter estimates, while including unimportant variables weakens the prediction or estimation capability of models. Many selection techniques exist in model building. First among them is the All-Subset Model (ASM) regression technique which requires the generation of all the possible combinations of the p-variables (Carley et al (2013)). The method guarantees that the best subset of variables is found, however, it is very computationally demanding and time consuming. As a consequence, the method becomes unsuitable for large numbers of variables.

Hastie et al (2009), used Sequential Search (SS), which is a simple method aimed at finding optimal subsets of variable for a specified model size. The basic idea is to replace each variable at a time with all the remaining ones and see whether a better model is obtained. The procedure of Sequential Search (SS) differs from All Subset Models method in that not all the possible combinations of the p-variables are tested. The method is thus being less consuming than ASM.

Carley et al (2013), used the stimulated annealing method of variable selection for searching the best subset of regressors. They stressed further, that this method is also useful as an automated tool for model calibration and validation especially for modern computational multi-agent large-scale social-networks systems. This is due to the fact that the large scale systems are heavily used in modeling and simulation of complex social networks and can as well be integrated in many large-scale simulation systems such as Biowar, etc.

2.2 Criterion-Based Procedures of variable selection

2.2.1 The Akaike Information Criterion (AIC)

The Akaike Information Criterion is a measure of the relative quality of a statistical model from a given set of data. It is called information theoretic, because it is based on Kullback-Leibler (K-L) information. (Burnham and Anderson (2002)) showed that K-L information theory represents information lost when model is used to approximate full realities. They considered the distance between approximated model and full reality and hence proposed that one would like to select a model from among R models that minimizes K-L information loss. This implies selecting the model from the set of models that loses the least information about full reality. The selected model is closest model to full reality in the set of the R models.

K-L information can be expressed as a difference between two expectations (Burnham and Anderson (2002)) and (Anderson and Burnham (2008)). The Akaike information criterion find a link between the K-L information and the maximized log-likelihood, the concept of likelihood is a fundamental to statistical theory [Azzalin (1996), Severini, (2010) and (Pawitan (2011)].

Operationally, the AIC is computed for each of the R models and selects the model with the smallest AIC value as best. Such a model is best in the sense of minimizing K-L information loss. Further details are in (Burnham and Anderson (2002)).

A second-order bias correction for AIC was derived by Reunanen, (2003) and is important to use in practice particularly when sample sizes are small. This criterion is denoted as AIC_c to make it distinct from AIC. The AIC penalizes for the addition of parameters, and thus selects a model that fits well but has a minimum number of parameters, (ie, simplicity and parsimony). For small sample sizes [ie, $\frac{n}{p} \leq 40$], he opines that the second-order Akaike Information Criterion (AIC_c) should be more appropriate than AIC. As sample size increases, the least of the AIC_c approaches zero, and the AIC_c tend to yield the same conclusions as the AIC (Burnham and Anderson (2002)).

The value of the AIC for a given data set has no meaning in itself. However meaningful, when it is compared to the AIC of a series of R models specified a priori, the model with the lowest AIC is the best model among all models specified for the data at hand (Burnham and Anderson (2002)). If only poor models are considered, the AIC will select the best of the poor models (Anderson and Burnham (2008)). The importance of determining the set of candidate models cannot be overemphasized. Hence, after having specified the set of models to explain the data, one should assess the fit of the full model (Burnham and Anderson (2002)). They argued further that, if the full models fit, simpler models also fit because they originated from the full model.

2.2.2 Bayesian Information Criterion (BIC)

The Bayesian Information Criterion (BIC) or Schwarz Criterion is a criterion for model selection among a finite set of models. The model with the lower BIC is preferred. It is based in part, on the likelihood function and is closely related to the Akaike Information Criterion (AIC). When fitting models, it is possible to increase the likelihood by adding parameters, but doing so may result in over-fitting. Both BIC and AIC resolve this problem by introducing a penalty term for the number of parameters in the model. The BIC was developed by Schwarz (1978).

2.3 Assessment of Adequacy of Regression Models

Many methods have been provided for assessing the goodness and adequacy of formulated models. They include the Adjusted R-square (R^2 adj), Predicted Residual Sum of Squares (PRESS), Mallow's C_p Statistic etc. The coefficient of determination, R^2, assumes that every independent variable in the model helps to explain variation in the dependent variable (y). It gives us the percentages of explained variation if all independent variables in the model affect the dependent variable (y). On the other hand $R^2{}_{adj}$ tells us the percentage of variation explained by only those independent variables that truly affect the dependent variable (y). Hence the value of $R^2{}_{adj}$ will be less than the value of R^2 (Nduka, (2010)). Neither the coefficient of determination nor the usual statistic F-test and prediction interval provide insight into the quality and potential influence of individual observations on the estimates.

The PRESS statistic, pioneered by Allen, (1974) can be used as a data validation procedure. The PRESS statistic is derived from the jackknifed residuals and it is generally known as the prediction sum of squares. It leaves out one observation, by fitting a given model to the rest of the data and then predicts the one left out and obtain the square of discrepancy. Also, it repeats this for all the single data points and omit the one that obtain the squares of discrepancies for a given model Allen, (1974).

The PRESS statistic is equivalent to the sum of studentized residuals, and can be extended to select parameters like the regularization weight on smoothing splines as shown by Sugiura, (1978)). The PRESS statistic does not depend on some particular model parameters, but on the model itself. As with techniques based on cross validation, it expresses to which extent a particular model is able to generate new data.

The PRESS statistic is complementary to the cost function. This cost function often expresses the discrepancy between measurements and the values predicted by the cost function. While minimizing the cost function allows one to find the model parameters, it is clear that the best model always has the lowest residuals error. The PRESS statistic is used as a measure of predictivity to compare and select the best model while minimizing the cost function, given the parameter of a particular model.

Hence, the best model will have a comparatively small prediction sum of squares. Mallow's C_p statistic was developed for choosing the number of covariates in a regression model. If model (p) is correct, then C_p will tend to be close to or smaller than p. Therefore, a plot of C_p versus p can be used to decide among models.

2.4 Reasons for Variable Selection in Model Building

The following reasons were suggested by Reunanen (2003) for selecting only a smaller subset of the variables to be included in a model instead of the whole set of candidate variables.

(i) That when modeling a particular property, it is reasonable to assume only a small number of variables are actually correlated with experimental response and are also relevant for building the mathematical model of interest.

(ii) That prediction/accuracy may be improved via exclusion of redundant or irrelevant variables.

(iii) That it is cheaper to measure only a reduced set of variables.

(iv) The models to be built are usually simpler and potentially faster when fewer input variables are included in the model.

(v) That knowing which variables are relevant can give insight into the nature of the prediction problem and allows a better understanding of the model.

3.0 Materials and Methods

The data used for this research study is as presented below:

3.1 Data Presentation

Money income, million currency units (y)	Inflation, monthly percent change in the CPI (X1)	Exchange rate, USD (X2)	Consumer Price Index (CPI) (X3)	Private sector credit, million currency units (X4)
2156.84	1.29	134.12	57.63	1489.35
2231.97	2.32	133.59	58.96	1524.25
2269.18	0.75	133.33	59.40	1543.11
2263.59	1.65	133.33	60.39	1534.45
2349.89	0.88	133.33	60.92	1607.46
2444.11	1.49	133.33	61.83	1623.28
2568.07	1.67	133.33	62.86	1666.76
2503.99	1.71	133.33	63.94	1722.90
2566.10	1.90	133.33	65.15	1789.15
2691.30	1.70	134.28	66.26	1840.77
2803.53	7. 42	135.14	71.18	1907.74
2775.69	2.48	135.99	72.94	1940.46
2772.99	-1.78	131.81	71.64	1986.21

2699.80	-2.39	130.95	69.93	1937.03
2664.64	-2.21	130.50	68.39	2031.57
2814.85	-1.48	130.00	67.37	2007.36
2841.03	0.10	129.87	67.44	1981.71
2877.49	1.61	129.16	68.53	1979.08
3003.13	2.77	128.49	70.43	1946.96
3179.83	2.19	128.45	71.97	2080.53
3566.53	0.08	128.41	72.02	2351.09
3733.72	-0.19	128.37	71.88	2303.70
3852.80	1.99	128.21	73.31	2384.80
3815.55	3.22	128.21	75.67	2542.81
3848.93	0.59	128.23	76.12	2571.68
4037.47	-2.50	128.19	74.22	2550.28
3844.59	-0.70	128.15	73.69	2577.50
3674.64	-0.76	128.37	73.13	2565.83
3920.46	-0.43	128.15	72.82	2608.92
3906.45	0.77	128.17	73.38	2885.51
3998.10	1.02	128.17	74.12	3003.78
3960.31	1.19	127.94	75.01	3188.10
4047.37	0.47	127.54	75.36	3414.41
4079.81	1.52	127.45	76.51	3447.83
4151.19	0.46	127.16	76.86	3829.25
5550.49	2.60	126.58	78.86	3909.45
5672.62	0.50	125.76	79.25	4203.17
5585.23	-2.09	123.64	77.60	4415.56
5881.56	-0.13	119.57	77.50	4712.72
5809.83	0.56	118.25	77.93	5056.72
6527.67	1.45	117.80	79.06	5338.43
7016.47	0.27	117.52	79.27	5739.40
7998.23	0.78	116.82	79.89	5964.33
7805.09	1.56	117.54	81.13	6494.21
7546.33	1.89	117.85	82.67	6787.46
7948.37	3.69	117.81	85.72	6754.68
8067.59	2.16	117.75	87.58	7341.12
8335.29	1.17	117.73	88.60	7425.09
8960.29	1.10	117.65	89.58	7474.67
8339.12	-0.60	117.78	89.04	7693.78
8387.16	-0.05	117.99	88.99	7973.27
9167.07	0.75	134.29	89.66	8059.55
9294.04	0.55	145.17	90.15	8508.58
9087.97	0.75	148.19	90.83	8467.98
8997.82	0.59	148.01	91.36	8226.44
9001.01	0.59	148.12	91.90	8379.91

8720.58	1.84	148.60	93.59	8509.35
9077.03	1.85	148.52	95.32	8556.94
8889.36	2.07	149.76	97.29	9026.07
9475.32	1.13	155.29	98.39	9675.02
9458.49	0.50	153.19	98.88	9811.36
9911.55	0.48	150.00	99.35	9853.62
10239.56	0.65	151.03	100.00	9989.98
10780.63	2.15	149.74	102.15	10219.34
10446.37	0.96	150.48	103.13	10082.00
10792.65	1.85	151.00	105.04	10069.46
11023.31	-0.14	150.13	104.90	10050.67
10972.49	0.79	150.35	105.72	10079.23
10759.31	-0.04	151.37	105.68	10026.97
10845.50	2.91	151.15	108.76	10102.82
10941.44	1.08	150.24	109.94	9910.71
11520.64	1.76	150.61	111.87	10113.20
11224.79	0.46	152.46	112.38	10336.11
11224.61	0.30	151.92	112.72	10534.37
11142.65	0.04	150.53	112.77	10748.38
11525.53	1.29	152.79	114.22	9830.34
11561.53	1.20	152.52	115.59	9416.71
11595.67	0.96	152.80	116.70	9178.29
11653.62	1.37	155.20	118.30	9446.95
11898.96	-0.54	154.55	117.66	9900.03
11986.23	0.91	156.11	118.73	9849.25
12172.10	0.97	155.57	119.89	9957.95
12389.27	0.32	152.43	120.27	9951.81
12508.01	1.67	153.79	122.27	10884.23
12618.08	1.41	156.66	124.00	11110.74
12172.50	0.49	159.66	124.60	11638.43
12210.41	0.04	158.78	124.65	12402.89
13303.49	1.06	162.42	125.97	14183.59
13755.29	3.35	161.33	130.19	14641.55
13153.79	0.28	158.59	130.55	14010.81
13270.97	1.59	157.65	132.63	14119.89
13304.78	0.13	157.44	132.80	14192.62
13603.14	0.75	158.42	133.80	14472.59
13483.06	1.15	162.39	135.34	14701.06
13392.43	0.24	161.34	135.66	14845.38
13770.06	0.67	158.72	136.57	14707.98
14065.27	1.01	157.74	137.95	14754.00
14399.30	0.88	157.20	139.17	14941.12
15062.73	0.60	157.52	140.01	15252.55

15483.85	0.75	157.17	141.06	15151.76
15308.39	0.62	156.89	141.94	14992.71
15547.62	0.75	157.43	143.00	15085.17
15669.17	0.71	158.33	144.02	15261.94
15634.38	0.55	158.16	144.82	15408.18
15424.05	0.67	158.01	145.79	15485.23
15593.17	0.59	160.21	146.65	15692.04
14811.43	0.54	161.14	147.44	15796.61
14619.45	0.25	161.08	147.81	16101.21
14362.45	0.75	161.96	148.92	16279.27
14529.51	0.75	159.84	150.04	16276.44
14734.88	0.72	158.71	151.11	16451.79
15688.96	0.78	159.03	152.29	16191.47
15493.69	0.64	160.08	153.26	16064.26
15424.18	0.50	163.81	154.03	16394.06
15738.91	0.78	164.75	155.23	16514.23
16041.03	0.62	162.16	156.19	16776.26
16156.59	0.78	161.81	157.41	16735.27
16171.62	0.77	162.89	158.62	16982.02
16669.20	0.65	162.22	159.65	17267.58
16375.44	0.48	162.00	160.42	17415.05
16814.45	0.55	163.05	161.31	17680.98
16558.08	0.51	164.73	162.13	17869.45
16696.66	0.59	171.82	163.09	18128.50
18927.79	0.82	182.79	164.44	18115.21
18975.00	0.81	185.83	165.77	18165.67
18871.26	0.68	198.39	166.90	18711.74
19142.53	0.91	199.30	168.42	18579.22
19267.59	0.76	199.01	169.71	18595.24
19193.98	1.10	199.06	171.58	18651.81
18811.43	0.93	198.94	173.17	18897.31
18424.70	0.69	198.94	174.37	18665.28
18491.57	0.59	199.14	175.40	18631.27
18718.00	0.61	199.13	176.46	18732.17
18204.40	0.42	199.16	177.20	19086.75
18367.24	0.66	199.12	178.37	18706.81
20029.83	0.99	199.10	180.15	18719.26
19799.46	0.87	199.13	181.71	18939.55
20620.80	2.30	199.01	185.89	19051.11
20470.44	2.17	199.06	189.94	18882.24
20727.91	1.61	199.04	192.99	19378.61
21035.13	2.75	199.05	198.30	19048.96
22078.01	1.71	216.40	201.70	21424.95

22535.19	1.25	293.11	204.23	22373.29
22057.00	1.01	323.20	206.29	22664.21
22013.78	0.81	315.03	207.96	22645.51
22180.07	0.83	315.41	209.68	23069.64
22386.05	0.78	315.27	211.33	23045.40
23591.73	1.06	315.07	213.56	21982.15
23096.53	1.01	312.59	215.72	21878.06
22210.95	1.49	314.39	218.95	22231.69
22304.27	1.72	313.52	222.71	22371.83
21768.24	1.60	309.29	226.27	22244.13
22047.77	1.88	315.21	230.53	22025.12
21980.58	1.58	317.47	234.17	21985.95
22195.02	1.21	314.87	237.02	22172.37
21851.45	0.97	361.95	239.32	21997.22
21953.99	0.78	358.74	241.19	22021.90
22500.62	0.76	359.06	243.03	21928.92
22311.12	0.78	358.77	244.93	21956.66
24140.63	0.59	359.54	246.38	22290.66
23963.03	0.80	359.80	248.35	21993.43
24143.01	0.79	360.53	250.32	22500.91
24424.42	0.84	359.76	252.41	22363.23
24474.15	0.83	359.98	254.52	22276.49
25169.01	1.09	360.36	257.29	22206.74
24814.00	1.24	360.22	260.47	22281.87
24971.10	1.13	360.56	263.42	22261.21
24859.35	1.05	361.29	266.18	22470.24
25560.66	0.84	361.99	268.41	22967.44
26041.90	0.74	363.27	270.39	23138.50
25467.43	0.80	362.98	272.56	23095.17
27068.58	0.74	363.81	274.57	22708.22
26771.09	0.74	363.01	276.60	22946.79
26546.64	0.73	361.81	278.62	24165.58
26834.82	0.79	361.02	280.81	23994.74
27579.45	0.94	360.02	283.46	24887.98
27813.70	1.11	360.34	286.61	24882.44
27898.83	1.07	354.10	289.69	24758.24
28277.14	1.01	360.91	292.62	24290.25
27556.80	0.99	362.44	295.51	24819.23
27669.96	1.04	361.99	298.59	25486.51
27811.79	1.07	361.90	301.78	25848.37
28415.37	1.02	362.37	304.87	26412.74
28783.19	0.85	362.81	307.47	26694.53
29132.38	0.87	362.51	310.16	26651.99

29719.36	0.79	364.40	312.61	26780.12
31010.12	0.84	369.40	315.23	28422.91
31958.48	1.02	386.94	318.45	28893.88
32719.16	1.17	387.69	322.17	29349.25
32593.52	1.21	386.20	326.07	29555.51
33337.33	1.25	387.13	330.14	30188.38
34262.59	1.34	386.06	334.57	30382.03
34965.45	1.48	384.78	339.52	29745.78
35729.18	1.54	382.46	344.73	29087.98
36517.80	1.60	383.44	350.26	29411.88
37704.98	1.61	386.11	355.91	30149.60

Source:2020 Statistical Bulletin of Nigeria Financial Sector Report and Central of Bank of Nigeria

3.2 Data Analysis

This section employed the methods of variable selection techniques, descriptive statistics and SPSS 25 statistical software in the analysis of the data on Money Income, Inflation rate, Exchange Rate, Consumer Price Index and Private Sector Credit obtained from time series monthly records of 2020 Statistical Bulletin of Nigerian Financial Sector Report and Central Bank of Nigeria. The methods of variable selection techniques are:

Forward Selection

Backward Selection

Stepwise Regression

Each of the methods is expected to generate a series of models assumed to adequately fit the Time Series data. All models generated will have an intercept, α_0. Measures of goodness shall be ascertained using standard assessment criteria. The $R^2, R^2_{adj}, PRESS$, p-values shall be employed in testing for the resulting models. Moreover, the D-optimality criterion shall be employed in assessing the goodness and adequacy of the models. Its performance shall be compared with those of the standard assessment criteria.

3.3 Model Formulation

From the data collected on Money Income (Y_i), Inflation Rate (X_1), Exchange Rate (X_2), Consumer Price Index (X_3), Private Sector Credit (X_4) and the number of parameters to be estimated, the full model for this study is presented as shown below:

$$\begin{aligned} Y_i = {} & \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_1 X_2 + \beta_6 X_1 X_3 + \beta_7 X_1 X_4 \\ & + \beta_8 X_2 X_3 + \beta_9 X_2 X_4 + \beta_{10} X_3 X_4 + \beta_{11} X_1^2 + \beta_{12} X_2^2 + \beta_{13} X_3^2 + \beta_{14} X_4^2 \end{aligned} \quad (3.0)$$

Where;

$\beta_0, \beta_1, \beta_2, \ldots\ldots\ldots, \beta_{14}$, are the coefficients of the model to be estimated.

3.3.1 Forward Selection

Forward selection method starts with a model of no parameters (except the intercept) and proceeds by adding variables that fulfill a defined criterion. Typically, the variable to be added at each step is the one that minimizes residual sum of squares (RSS) at most. This can be evaluated also by F-test, defined by: $F_j = max_j \left(\frac{RSS_p - RSS_{p-j}}{S^2{}_{p-j}}\right) > F_{in} = F(\alpha)_{p,n-p}$ (3.1)

Where;

RSS_p and RSS_{p-j} are the residuals sum of squares of the models with p and p-j variables respectively.

$S^2{}_{p-j}$ is the variance of the model built with p-j variables

F_{in} is used as a stop criterion

The sequential steps of Forward Selection method are:

Start with no variable in the model other than the intercept:

For all predicators not in the model, check if their p-values are added to the model:

Choose the one with lowest p-value less than α-critical:

Continue until no new predicators can be added.

3.3.2 Backward Selection

Unlike the forward selection, the backward elimination proceeds in the opposite direction. It starts from a model of size p, p being the total number of variables, and eliminates variables that are not relevant in a step by step procedure. The variable to be deleted is usually the one that gives the minimum increase in RSS. Like in forward selection method, this can be evaluated by an F-test, defined by; $F^{-}{}_j = max_j \left(\frac{RSS_{p-j} - RSS_p}{S^2{}_{p-j}}\right) > F_{out} = F(\alpha)_{p,n-p}$ (3.2)

Where;

RSS_{p-j} and RSS_p are the residuals sum of squares of the models with p-j and p variables respectively.

$S^2{}_p$ is the variance of the model built with p variables;

F_{out} is used as stop criterion

The sequential steps of Backward Elimination are;

Start with all the predicators in the model;

Remove the predicators with highest p-value greater than α-critical;

Refit the model and return to (ii);

Stop when all p-values are less than α-critical.

3.3.3 Stepwise Regression Method

The two procedures described above suggest a number of possible combinations. One of the most popular is the Stepwise Regression algorithm. This is the modification of the Forward Selection method in which at each step all regressors entered into the model are reassessed via their partial F- or t-statistic. A regressor added at an earlier step may now be redundant because of the relationship between it and the regressors now in the model. If the partial F-statistic for a variable is less than F_{out}, that variable is dropped from the model. However, the variable may be brought in again at a later stage. The selection stops when all unused variables are not significant and all the included variables are significant.

Stepwise regression requires two cut-off values F_{in} and F_{out}. Some analyst prefer to choose $F_{in} = F_{out}$, although this is not necessary. Sometimes $F_{in} > F_{out}$ is chosen, making it more difficult to add a regressor or delete one.

3.3.4 Akaike Information Criterion (AIC)

The general equation for calculating AIC is:

$$AIC = -2 * ln(Likelihood) + 2 * P \quad (3.3)$$

Where;

ln is the natural logarithm,

$(Likelihood)$ is the value of the likelihood,

P is the number of parameters in the model.

AIC can be calculated using residual sum of squares from regression.

$$AIC = n * ln\left(\frac{RSS}{n}\right) + 2 * P \quad (3.4)$$

Where;

n is the number of data points (observations),

RSS is the residual sum of squares.

AIC requires a bias-adjustment for small sample sizes. If ratio of $\frac{n}{p} \leq 40$, then we shall use bias-adjustment:

$$AIC_c = n * ln(likelihood) + 2P + \frac{(2p(p+1))}{(n-p-1)} (3.4)$$

where, variables are as defined above. As the size of the data set n increase relative to the number of parameters, p, the bias-adjustment term on the right becomes very small. Therefore, it is recommended that we always use the small sample adjustment.

3.3.5 R-Squared (R^2)

R^2 is a measure of the proportion of variability in the data set as accounted for by a regression model. It assumes that every independent variable in the model helps to explain variation in the dependent variable (y) and thus gives the percentage of explained variation if all independent variables in the model affect the dependent variable (y). The R^2 statistic is defined as:

$$R^2 = 1 - \frac{SSR}{SST} (3.5)$$

Where;

$SST = (y_i - \bar{y})^2$ is the total sum of squares,

$SSR = \sum_i^n (\hat{Y}_i - \bar{y})^2$ is the regression sum of squares,

y_i and $\bar{y}_i$ are the original and modeled data values.

3.3.6 Adjusted R-Squared ($R^2{}_{adj}$)

In least squares regression, increasing the number of regressors in the model leads to increase in R^2. Hence R^2 alone cannot be employed as a meaningful comparism of models. The adjusted R-Square $(R^2{}_{adj})$, tells us the percentage of variation explained by only those independent variables that truly affect the dependent variable (y) and penalizes for adding independent variable(s) that do not belong to the model. The adjusted R-Square $(R^2{}_{adj})$ is defined as:

$$R^2{}_{adj} = 1 - \frac{\frac{SSR}{(n-p)}}{\frac{SST}{(n-1)}}$$

$$= 1 - \frac{n-1}{n-p}(1-R^2)(3.6)$$

Where;

n is the sample size,

p is the model parameters,

$SSR = \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2$ is the regression sum of squares,

$SST = \sum_{i=1}^{n}(y_i - \bar{y})^2$ is the total sum of squares.

3.3.7 Predicted Residual Sum of Squares (PRESS)

The Predicted Residual Sum of Squares (PRESS) provides a useful residual scaling. It serves as data validation procedure. It provides an insight into the quality and potential influence of individual observation on the estimates. It is defined as:

$$PRESS = \frac{\sum_{i=1}^{n}(\dot{y}_i - \hat{y}_i)}{[1 - x_i(X'X)^{-1}x'_i]^2}(3.7)$$

Where;

y_i is the dependent variable,

$\hat{y}$ is the estimated dependent variable which the regression model is fitted,

$(X'X)^{-1}$ is the variance-covariance matrix,

x_i is the column vector of independent variable

x'_i is the row vector of independent variable.

Note that, the smaller the PRESS value, the better the model's predictive ability.

4.0 Results and Discussion of Findings

4.1 Descriptive Statistics

This section deals with data presentation, analysis and discussion of results. The data used in this research were collected from 2020 Statistical Bulletin of Nigeria Financial Sector Report and Central of Bank of Nigeria Bulletin. The data sets were monthly observations from 2004 to 2020; a total of 196 observations for each variable. The dependent variable (Y_i) is Money income (million currency units), while the independent variables are *Inflation* rate (monthly percent change in the CPI "X_1"); *Exchange rate*, (USD "X_2"); *Consumer Price Index* (CPI "X_3") and *Private Sector Credit* (PSC billion currency units "X_4"). The descriptive statistics are summarized on Table 4.1.

Table 4.1: Descriptive Statistics of the variables used

Variables	Minimum	Maximum	Mean	Std. Deviation
Money income, (million currency units y)	2156.84	37704.98	15099.96	8856.49
Inflation, (monthly percent change in the CPI X_1)	-2.50	7.42	0.944	0.988
Exchange rate, (USD X_2)	116.82	387.69	207.04	93.86
Consumer Price Index (CPI X_3)	57.63	355.91	157.7579	80.66
Private sector credit, (million currency units X_4)	1489.35	30382.03	14149.5817	8305.67

Table 4.1 shows the descriptive statistics for Inflation rate having negative value at its minimum value and a light positive at its maximum value, while the other variables are all strongly positive at both minimum and maximum values. These results suggest that Inflation rate of Nigeria can be as lower than zero.

4.2 Regression Models and their Parameter Estimates (Co-efficient)

This section deals with the actualization of the regression models discussed in section three; which are the full regression model, that is the stepwise selection method model, backward selection method model and forward selection method model. This estimation was achieved using SPSS 25 statistical software and the results of data analysis are fully reported.

4.2.1 Full Regression Model

The full regression model is expressed as

$$Y_i = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_1 X_2 + \beta_6 X_1 X_3 + \beta_7 X_1 X_4 + \beta_8 X_2 X_3 + \beta_9 X_2 X_4 + \beta_{10} X_3 X_4 + \beta_{11} X_1^2 + \beta_{12} X_2^2 + \beta_{13} X_3^2 + \beta_{14} X_4^2 \quad (4.1)$$

Recall that, β_0, β_1, β_2, ..., β_{14} are the model estimated parameters (coefficients). The full models have square in the interaction of the independent variables (a total of 15 parameters were estimated).

Table 4.2: Full Regression Model with its Estimated Parameters

Parameters	Unstandardized Coefficients B	Std. Error	t-test	(p-value) Sig.
(Constant)	**-1248.116**	**1023.247**	**-1.220**	**0.224**
Inflation rate (X_1)	**61.070**	**279.153**	**0.219**	**0.827**
Exchange rate, (X_2)	39.000	13.637	2.860	0.005**
Consumer Price Index (CPI) (X_3)	**-2.501**	**17.119**	**-0.146**	**0.884**
Private sector credit, (X_4)	0.717	0.102	7.061	0.000**
X_1^2	**-7.004**	**15.398**	**-0.455**	**0.650**
X_2^2	-0.169	0.038	-4.389	0.000**
X_3^2	**0.456**	**0.234**	**0.809**	**0.419**
X_4^2	-3.190E-5	0.000	-7.139	0.000**
X_1X_2	-19.119	4.353	-4.392	0.000**
X_1X_3	39.412	7.917	4.978	0.000**
X_1X_4	-0.150	0.038	-3.928	0.000**
X_2X_3	**-0.047**	**0.255**	**-0.062**	**0.951**
X_2X_4	0.003	0.001	4.103	0.000**
X_3X_4	0.001	0.001	2.130	0.035**

Footnote: Dependent Variable: Money income, (y); **= Sig. at 0.05

Table 4.3: R-square, Adjusted R-Square, MSE and ANOVA

Model	R Square	Adjusted R Square	Mean Square Error	ANOVA $F_{cal,}$ (p-values)
Full Regression Model	0.995	0.995	425693.261	2978.947 (0.000^{**})

Footnote: **= Sig. at 0.05

Table 4.3 shows that the ANOVA p-value for the full regression model is (0.000) and indicates that the model estimated by the regression procedure is significant at an α level of 0.05. This implies that at least one coefficient is different from zero. Also, the R-squared value indicates that

the predictors explain 95.5% of the variance in Money income. The adjusted R-square is 99.5%, which accounts for the number of predictors in the model. Both (R-square and adjusted R-square) values indicates that the model fits the data well. The model does not appear to be over-fitted and has adequate predictive ability.

In Table 4.2, out of the 15 fitted parameters, 6 estimated parameters ($\beta_0 = 0.224\,, \beta_1 = 0.827\,, \beta_3 = 0.884\,, \beta_{11} = 0.650\,,\ \beta_{13} = 0.419\ \ and\ \ \beta_8 = 0.951$) are not significant, at an α level of 0.05. The p-values for the estimated coefficients of Exchange rate and Private sector credit with their interaction and square ($\beta_2 = 0.005\,, \beta_4 = 0.000\,, \beta_{12} = 0.000\,, \beta_{14} = 0.000\ and\ \beta_9 = 0.000$) are less than 0.05, indicating that they are significantly related to Money income. While the p-value for Inflation rate and Consumer Price Index are with their interactions and square greater than 0.05 (not significant), indicating that it is not related to Money income at an α level of 0.05.

Additionally, it indicates that the predictor Inflation rate and Consumer Price Index with their interactions and square doesn't explain a substantial amount of unique variance in Money income. Only the interaction between Exchange rate and Consumer Price Index ($\mathbf{X_2X_3}$) are not significant. However, this result suggests that a model with only Exchange rate and Private sector credit with their interactions and square may be more appropriate.

4.2.2 Stepwise Selection method Regression models

The stepwise selection method was used to identify the best model from the several regression models built. This method identifies seven suitable models for the prediction of relationship between the dependent variable (Money income) and independent variables (Inflation rate, Exchange rate, Consumer Price Index and Private sector credit). That is

Table 4.4: Model Selection Criteria of the Stepwise Selection Method Models

Stepwise Selection Method Models	R Square	Adjusted R Square	ANOVA $F_{cal,}$ (p-values)	Mean Square Error	k	AIC
1	0.974	0.974	7408.595 (0.000**)	2011854.7	2	2848.86
2	0.991	0.991	11208.624 (0.000**)	676477.2	3	2637.23

3	0.993	0.993	8674.179 (0.000**)	589466.9	4	2612.25
4	0.993	0.993	7024.738 (0.000**)	540662.5	5	2597.31
5	0.994	0.993	5838.298 (0.000**)	520576.7	6	2591.89
6	**0.994**	**0.994**	**5102.761 (0.000**)**	**496511.4**	**7**	**2584.61**
7	0.994	0.994	4534.196 (0.000**)	479065.9	8	2579.60

Footnote: k is the number of parameters in the model.

Table 4.5: Stepwise Selection Method Models with their estimated Parameters

Stepwise Selection Method Models	Parameters	Unstandardized Coefficients B	Std. Error	t-test	p-value(Sig.)
1	**(Constant)**	**205.731**	**200.519**	**1.026**	**0.306**
	Private sector credit (X_4)	1.053	0.012	86.073	0.000**
2	(Constant)	1497.691	133.667	11.205	0.000**
	Private sector credit (X_4)	0.777	0.016	49.378	0.000**
	$\mathbf{X_3^2}$	0.083	0.004	19.595	0.000**
3	(Constant)	735.885	183.521	4.010	0.000**
	Private sector credit (X_4)	0.937	0.032	29.362	0.000**
	$\mathbf{X_3^2}$	0.129	0.009	14.229	0.000**
	X_4^2	-1.096E-5	0.000	-5.636	0.000**
4	(Constant)	681.545	177.176	3.847	0.000**
	Private sector credit (X_4)	0.929	0.031	30.164	0.000**
	$\mathbf{X_3^2}$	0.130	0.009	14.852	0.000**
	X_4^2	-1.182E-5	0.000	-6.276	0.000**
	X_1X_4	0.028	0.007	4.025	0.000**
5	(Constant)	751.358	175.521	4.281	0.000**
	Private sector credit (X_4)	0.931	0.030	30.790	0.000**
	$\mathbf{X_3^2}$	0.147	0.011	14.008	0.000**
	X_4^2	-1.205E-5	0.000	-6.514	0.000**
	X_1X_4	0.025	0.007	3.715	0.000**
	X_2^2	-0.011	0.004	-2.893	0.004**
6	(Constant)	1517.195	294.675	5.149	0.000**
	Private sector credit (X_4)	0.851	0.039	22.053	0.000**
	$\mathbf{X_3^2}$	0.137	0.011	12.729	0.000**
	X_4^2	-2.119E-5	0.000	-6.264	0.000**

	X_1X_4	0.022	0.007	3.295	0.001**
	X_2^2	-0.077	0.021	-3.653	0.000**
	X_2X_4	0.002	0.001	3.195	0.002**
7	**(Constant)**	**221.187**	**544.854**	**0.406**	**0.685**
	Private sector credit (X_4)	0.569	0.107	5.295	0.000**
	X_3^2	0.060	0.029	2.056	0.041**
	X_4^2	-2.483E-5	0.000	-6.961	0.000**
	X_1X_4	0.020	0.007	2.986	0.003**
	X_2^2	-0.138	0.030	-4.605	0.000**
	X_2X_4	0.003	0.001	4.297	0.000**
	Consumer Price Index (CPI) (X_3)	42.766	15.232	2.808	0.006**

Footnote: **= Sig. at 0.05.

From Table 4.4 and 4.5; the Stepwise Selection Method model fitted parameters that are significant at an α level of 0.05, except in Model 1 and 7; where the constant coefficient which represented other variables not included in this research is not significant at an α level of 0.05.

Hence, **Model 6** having the highest number of parameters estimate amongst the models is selected as the best since all its coefficients, are significant at an α level of 0.05.

$$Y_i = \beta_0 + \beta_4 X_4 + \beta_7 X_1 X_4 + \beta_9 X_2 X_4 + \beta_{12} X_2^2 + \beta_{13} X_3^2 + \beta_{14} X_4^2 \quad (4.2)$$

By substitution:

$$Y_i = 1517.20 + 0.851 X_4 + 0.022 X_1 X_4 + 0.002 X_2 X_4 - 0.077 X_2^2 + 0.137 X_3^2 - 2.11E^{-5} X_4^2 \quad (4.3)$$

4.2.3 Backward Selection Method Regression Models

Similarly, the backward selection method was used to identify the best model from the several regression models built. This method identified four suitable models for the prediction of the relationship between the dependent variable (Money income) and independent variables (Inflation rate, Exchange rate, Consumer Price Index and Private sector credit).

Table 4.6: Model Selection Criteria of the Backward Selection Method Models

Backward Selection Method Models	R Square	Adjusted R Square	ANOVA $F_{cal,}$ (p-values)	Mean Square Error	k	AIC
1	0.995	0.995	2978.947 (0.000**)	425693.3	13	2566.45
2	0.995	0.995	3267.136 (0.000**)	423429.1	12	2563.40
3	0.995	0.995	3612.512 (0.000**)	421241.1	11	2560.37
4	**0.995**	**0.995**	**4030.579 (0.000**)**	**419495.6**	**10**	**2557.57**

Footnote: k is the number of parameters in the model.

Table 4.7: Backward Selection Method Models with their estimated parameters

Backward Selection Method Models	Parameters	Unstandardized Coefficients		t-test	p-value (Sig.)
		B	Std. Error		
1	**(Constant)**	**-1248.116**	**1023.247**	**-1.220**	**0.224**
	Inflation, (X_1)	**61.070**	**279.153**	**.219**	**0.827**
	Exchange rate, (X_2)	39.000	13.637	2.860	0.005
	Consumer Price Index (CPI) (X_3)	**-2.501**	**17.119**	**-.146**	**0.884**
	Private sector credit, (X_4)	0.717	0.102	7.061	0.000
	X_1^2	**-7.004**	**15.398**	**-.455**	**0.650**
	X_2^2	-0.169	0.038	-4.389	0.000
	X_4^2	-3.190E-5	0.000	-7.139	0.000
	X_1X_2	-19.119	4.353	-4.392	0.000
	X_1X_3	39.412	7.917	4.978	0.000
	X_1X_4	-0.150	0.038	-3.928	0.000
	X_2X_4	0.003	0.001	4.103	0.000
	X_3X_4	0.001	0.001	2.130	0.035
2	**(Constant)**	**-1247.092**	**1020.498**	**-1.222**	**0.223**
	Inflation, (X_1)	**58.300**	**277.766**	**.210**	**0.834**
	Exchange rate, (X_2)	37.989	11.716	3.242	0.001
	Private sector credit, (X_4)	0.705	0.051	13.863	0.000
	X_1^2	**-7.359**	**15.165**	**-.485**	**0.628**
	X_2^2	-0.170	0.037	-4.539	0.000
	X_4^2	-3.165E-5	0.000	-7.685	0.000
	X_1X_2	-18.851	3.936	-4.789	0.000

	X_1X_3	38.916	7.134	5.455	0.000
	X_1X_4	-0.148	0.036	-4.160	0.000
	X_2X_4	0.003	0.001	5.024	0.000
	X_3X_4	0.001	0.000	3.333	0.001
3	**(Constant)**	**-1176.125**	**960.365**	**-1.225**	**0.222**
	Exchange rate, (X_2)	37.183	11.041	3.368	0.001
	Private sector credit, (X_4)	0.709	0.045	15.736	0.000
	X_1^2	**-7.237**	**15.114**	**-.479**	**0.633**
	X_2^2	-0.168	0.036	-4.616	0.000
	X_4^2	-3.159E-5	0.000	-7.711	0.000
	X_1X_2	-18.405	3.304	-5.570	0.000
	X_1X_3	38.968	7.111	5.480	0.000
	X_1X_4	-0.150	0.034	-4.404	0.000
	X_2X_4	0.003	0.001	5.064	0.000
	X_3X_4	0.001	0.000	3.335	0.001
4	**(Constant)**	**-1195.689**	**957.506**	**-1.249**	**0.213**
	Exchange rate, (X_2)	37.320	11.015	3.388	0.001
	Private sector credit, (X_4)	0.710	0.045	15.784	0.000
	X_2^2	-0.169	0.036	-4.664	0.000
	X_4^2	-3.186E-5	0.000	-7.868	0.000
	X_1X_2	-18.470	3.295	-5.606	0.000
	X_1X_3	38.714	7.077	5.471	0.000
	X_1X_4	-0.148	0.034	-4.388	0.000
	X_2X_4	0.003	0.001	5.127	0.000
	X_3X_4	0.001	0.000	3.379	0.001

Footnote: **= Sig. at 0.05.

In table 4.6 and 4.7, the Backward Selection Method fitted Model 1 showed that four parameters were not significant at an α level of 0.05; Model 2 showed that three parameters were not significant at an α level of 0.05; Model 3 showed that two parameters were not significant at an α level of 0.05 and Model 4 showed that one parameter was not significant at an α level of 0.05. The (constant coefficient); which represented other variables not included in this research is not significant at an α level of 0.05.

Hence, **Model 4** is selected as the best since all its coefficients are significant at, an α level of 0.05.

$$Y_i = \beta_0 + \beta_2 X_2 + \beta_4 X_4 + \beta_5 X_1 X_2 + \beta_6 X_1 X_3 + \beta_7 X_1 X_4 + \beta_9 X_2 X_4 + \beta_{10} X_{33} X_4 + \beta_{12} X_2^2 + \beta_{14} X_4^2$$

(4.4)

By substitution:

$$Y_i = -1195.09 + 37.32 X_4 + 0.71 X_4 - 18.47 X_1 X_2 + 38.71 X_1 X_3 - 0.15 X_1 X_4 + 0.003 X_2 X_4 + 0.001 X_{33} X_4 - 0.169 X_2^2 - 3.186 E^{-5} X_4^2 \quad (4.5)$$

4.2.4 Forward Selection method Regression models

Likewise, the forward selection method was used to identify the best model from the several regression models built. This method, also identified seven suitable models for the prediction of relationship between the dependent variable (Money income) and independent variables (Inflation rate, Exchange rate, Consumer Price Index and Private sector credit).

Table 4.8: Model Selection Criteria of the Forward Selection Method Models

Forward Selection Method Models	R Square	Adjusted R Square	ANOVA $F_{cal,}$ (p-values)	Mean Square Error	k	AIC
1	0.974	0.974	7408.595 (0.000**)	2011854.7	2	2848.86
2	0.991	0.991	11208.624 (0.000**)	676477.2	3	2637.23
3	0.993	0.993	8674.179 (0.000**)	589466.9	4	2612.25
4	0.993	0.993	7024.738 (0.000**)	540662.5	5	2597.31
5	0.994	0.993	5838.298 (0.000**)	520576.7	6	2591.89
6	0.994	0.994	**5102.761 (0.000**)**	**496511.4**	**7**	**2584.61**
7	0.994	0.994	4534.196 (0.000**)	479065.9	8	2579.60

Footnote: k is the number of parameters in the model.

Table 4.9: Forward Selection Method Models with their estimated parameters

Forward Selection Method Models	Parameters	Unstandardized Coefficients		t-test	p-value (Sig.)
		B	Std. Error		
1	**(Constant)**	**205.731**	**200.519**	**1.026**	**0.306**
	Private sector credit (X_4)	1.053	0.012	86.073	0.000**
2	(Constant)	1497.691	133.667	11.205	0.000**
	Private sector credit (X_4)	0.777	0.016	49.378	0.000**
	$\mathbf{X_3^2}$	0.083	0.004	19.595	0.000**

3	(Constant)	735.885	183.521	4.010	0.000**
	Private sector credit (X_4)	0.937	0.032	29.362	0.000**
	X_3^2	0.129	0.009	14.229	0.000**
	X_4^2	-1.096E-5	0.000	-5.636	0.000**
4	(Constant)	681.545	177.176	3.847	0.000**
	Private sector credit (X_4)	0.929	0.031	30.164	0.000**
	X_3^2	0.130	0.009	14.852	0.000**
	X_4^2	-1.182E-5	0.000	-6.276	0.000**
	X_1X_4	0.028	0.007	4.025	0.000**
5	(Constant)	751.358	175.521	4.281	0.000**
	Private sector credit (X_4)	0.931	0.030	30.790	0.000**
	X_3^2	0.147	0.011	14.008	0.000**
	X_4^2	-1.205E-5	0.000	-6.514	0.000**
	X_1X_4	0.025	0.007	3.715	0.000**
	X_2^2	-0.011	0.004	-2.893	0.004**
6	(Constant)	1517.195	294.675	5.149	0.000**
	Private sector credit (X_4)	0.851	0.039	22.053	0.000**
	X_3^2	0.137	0.011	12.729	0.000**
	X_4^2	-2.119E-5	0.000	-6.264	0.000**
	X_1X_4	0.022	0.007	3.295	0.001**
	X_2^2	-0.077	0.021	-3.653	0.000**
	X_2X_4	0.002	0.001	3.195	0.002**
7	**(Constant)**	**221.187**	**544.854**	**0.406**	**0.685**
	Private sector credit (X_4)	0.569	0.107	5.295	0.000**
	X_3^2	0.060	0.029	2.056	0.041**
	X_4^2	-2.483E-5	0.000	-6.961	0.000**
	X_1X_4	0.020	0.007	2.986	0.003**
	X_2^2	-0.138	0.030	-4.605	0.000**
	X_2X_4	0.003	0.001	4.297	0.000**
	Consumer Price Index (CPI) (X_3)	42.766	15.232	2.808	0.006**

Footnote: **= Sig. at 0.05.

The Forward Selection Method model results in Table 4.8 and 4.9 showed similar result as the Stepwise Selection Method fitted models, where all the parameters are significant at an α level of 0.05, except in Model 1 and 7. That is the constant coefficient which represented other variables not included in this research is not significant, at an α level of 0.05.

Hence, **Model 6** is selected as the best since all its coefficients are significant, at an α level of 0.05.

$$Y_i = \beta_0 + \beta_4 X_4 + \beta_7 X_1 X_4 + \beta_9 X_2 X_4 + \beta_{12} X_2^2 + \beta_{13} X_3^2 + \beta_{14} X_4^2 \quad (4.6)$$

By substitution:

$$Y_i = 1517.20 + 0.851 X_4 + 0.022 X_1 X_4 + 0.002 X_2 X_4 - 0.077 X_2^2 + 0.137 X_3^2 - 2.11E^{-5} X_4^2 \quad (4.7)$$

Table 4.10: Identification of Model Summaries with the number of estimated parameters

Model	**Total Parameters to be Fitted**	**Number of Significant Parameters Fitted**	**Remark**
Full Regression Model (FRM)	15	9	The same as BSMRM
Stepwise Selection Method RegressionModel (SSMRM)	15	7	The same as FSMRM
Backward Selection Method RegressionModel (BSMRM)	15	9	The same as FRM
Forward Selection Method RegressionModel (FSMRM)	15	7	The same as SSMRM

Table 4.10 showed that Full Regression Model (FSMRM) and Backward Selection Method Regression Model (BSMRM) have nine significant parameters estimated, while the Stepwise Selection Method Regression Model (SSMRM) and Forward Selection Method Regression Model (FSMRM) have seven significant parameters estimated.

Consumer Price Index (CPI) (X_3) and Private sector credit, (X_4) have significant effect on money income, from the identified model of SSMRM and FSMRM. Also, Exchange rate, (X_2) and Private sector credit, (X_4) have significant effect on money supply, from the identified model of FSMRM and BSMRM. Note that only inflation rate variable doesn't explain a substantial amount of unique variance in Money income.

Hence, the SSMRM and FSMRM are the best model as they identified the same model and the same number of significant parameters.

5.0 Conclusion and Recommendation

5.1 Conclusion

This result suggests that a model with only Exchange rate and Private sector credit with their interactions and square may be more appropriate or suitable model for estimating the behaviour of small-scale business Money income.

Recommendation

This research study recommends that: Firstly, Exchange rate and Private sector credit with their interactions and square of these variables may be appropriate for the prediction of the behaviour of small-scale business Money income. Secondly, that the Forward and stepwise selection method regression models may be the appropriate models for money income in Aba and finally, the study recommends that the exchange rate figure should be constant while the private sector credit should be doubled (to encourage private sector investment) as they have significant effect on money income of small-scale business in Aba Nigeria.

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